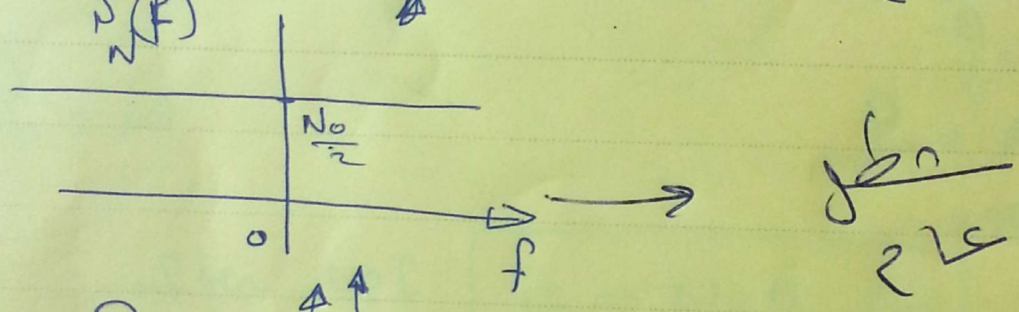
Ch 1 (Noise)

Noise: is any unwanted signal that disturb the transmission of signals.

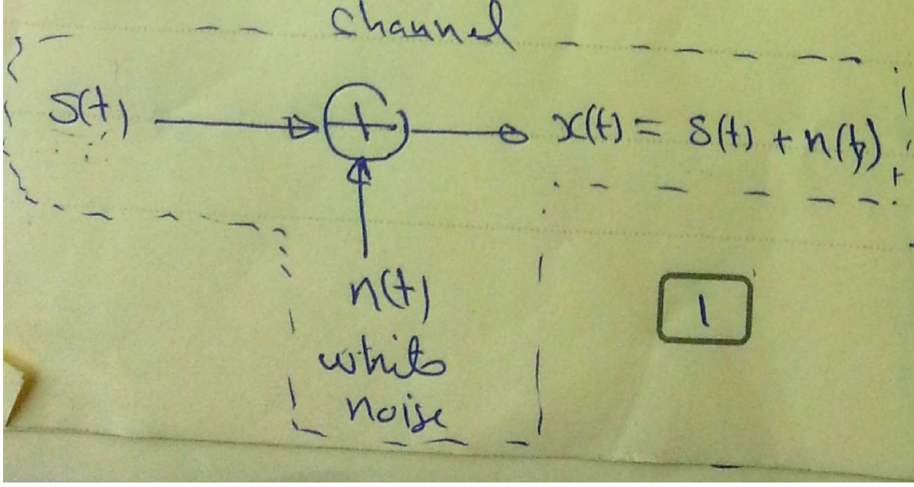
وأشهر تلك الأنواع هو ال white noise والتي
يتم إضافتها إلى ال channel in Comm. system وتكون شكل توزيع
ال white noise مع ال fr ثابتة



"Power spectrum density" of white noise "PSD"

Additive noise channel

ويتم تمثيلها بالشكل التالي حيث أنه يتم إضافة ال white noise إلى ال
channel لذا هو تأثير ال noise ع ال channel وال system

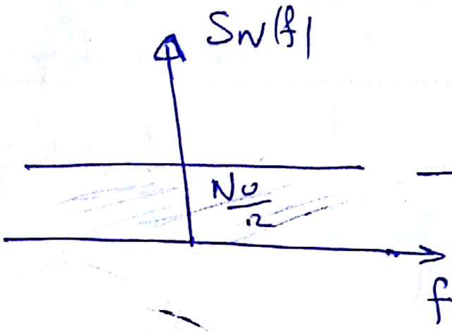


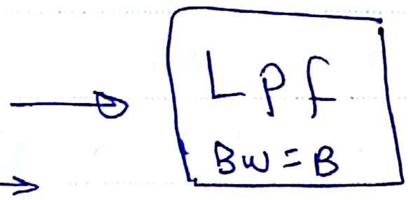
لذا أسمي ال white noise ب
additive white noise

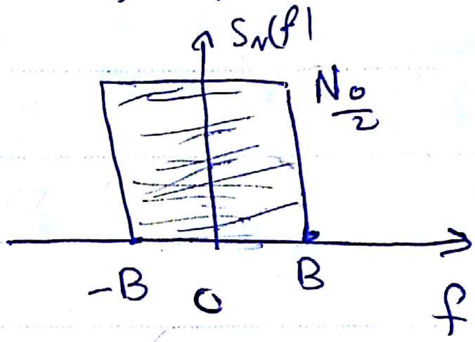
(I) (Ideal Low-Pass white noise)

n(t) → LPF → o/p

الهدف من ال filter هو تقليل
تأثير ال noise عن طريق تقليل
ال power الموجودة به







$P_n = \infty$

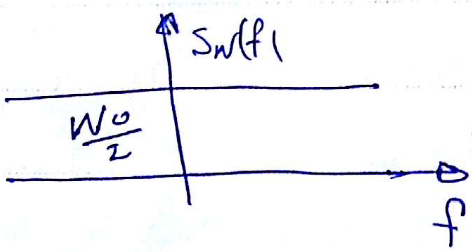
$P_n = \text{Area}$

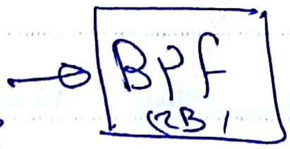
$= 2B \times \frac{N_o}{2} = N_o B$

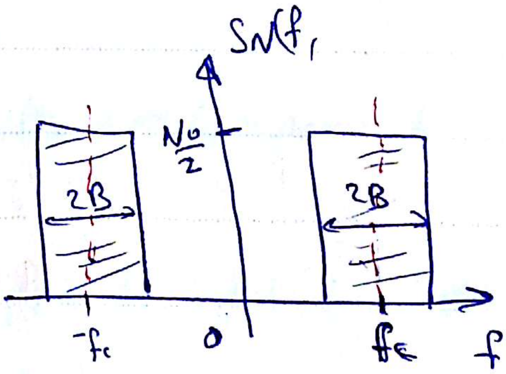
$\therefore$ after LPF $\boxed{P_n = N_o B \neq \infty}$

(II) (Bandpass noise ((Narrow band noise)))

n(t) → BPF (BW = 2B) → o/p







$P_n = \infty$

$P_n = \frac{N_o}{2}[2B + 2B]$

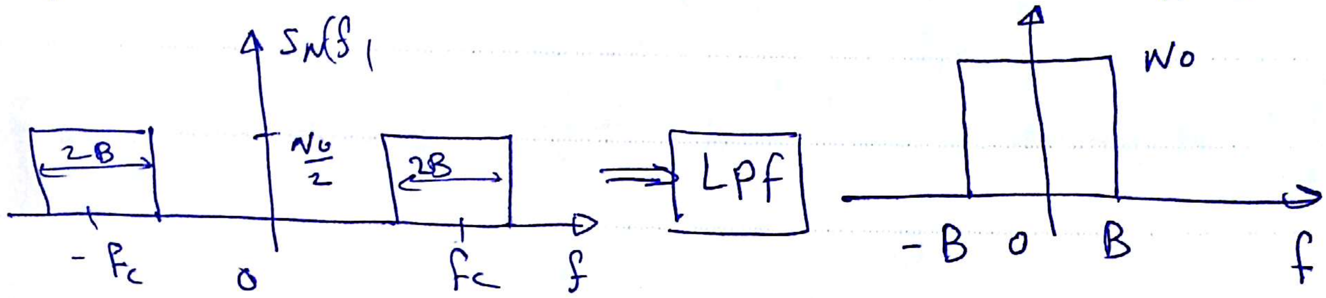
$\boxed{P_n = 2N_o w}$ $\boxed{B = w}$

$\boxed{P_n = 2N_o B}$

II (III) Ideally filtered narrow band noise

وهذا الشكل مهم جداً في عملية الـ DeModulation في الـ Rx

حيث تم إدخال الـ noise (Bpf) على Lpf كما بالشكل التالي



$$P_n = (2B+2B)\frac{N_o}{2} = 2N_o w$$

$$P_n = 2BN_o = 2N_o w$$

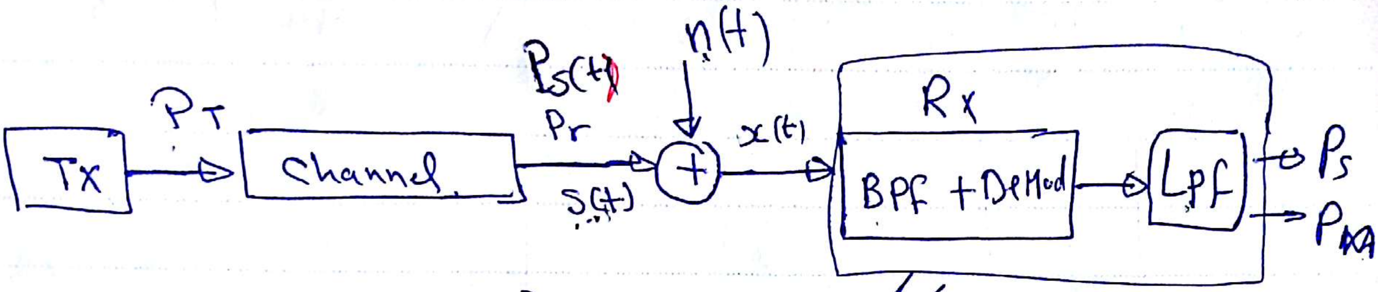
ملحوظة (1) عند التعامل مع الـ n(t) في الـ channel
يتم التعامل عن طريق المعادلة التالي

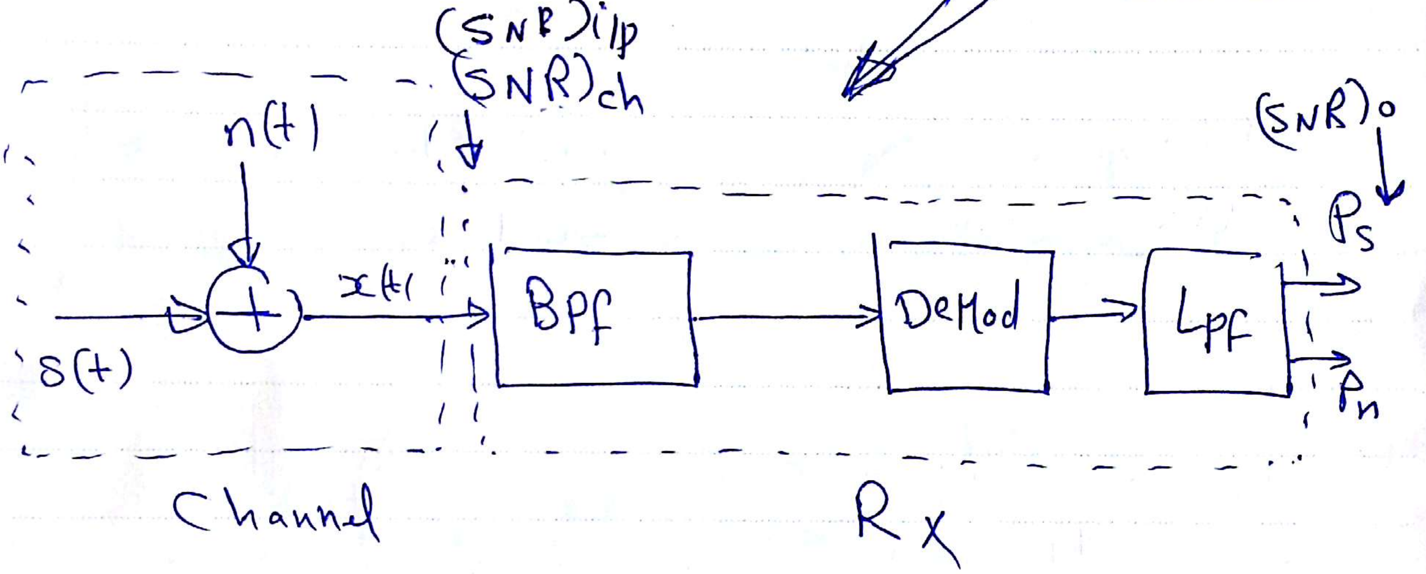
$$n(t) = \underbrace{n_r(t)\cos 2\pi f_c t}_{\text{In phase Component}} - \underbrace{n_Q(t)\sin(2\pi f_c t)}_{\text{Quadrature Component}}$$

ملحوظة (2) الجزء القادم الهدف منه حساب الـ noise عند دخل وخرج الـ Reciever لمعرفة مدى جودة الـ Rx ولكي نستطيع أن نفاضل بين الأنواع المختلفة

الصلح

Noise in C.S





* الهدف من أي نوع " AM - DSBSC - SSB - FM " هو

معرفة نسبة ال $(SNR)_{ch}$ وال $(SNR)_o$

$(SNR)_o \downarrow$ at Rx o/p

$(SNR)_{ch} \downarrow$ at Rx i/p

where

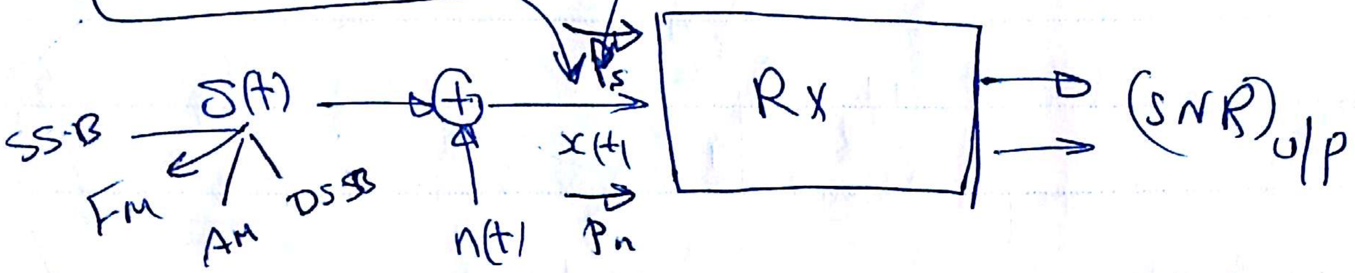
$$SNR = \frac{\text{average signal power}}{\text{average noise power}}$$

* ثم يتم حساب النسبة بينهم لحساب ال figure of Meriet

$$\text{Figure of Merit} = \frac{(SNR)_o}{(SNR)_{ch}} = FOM$$

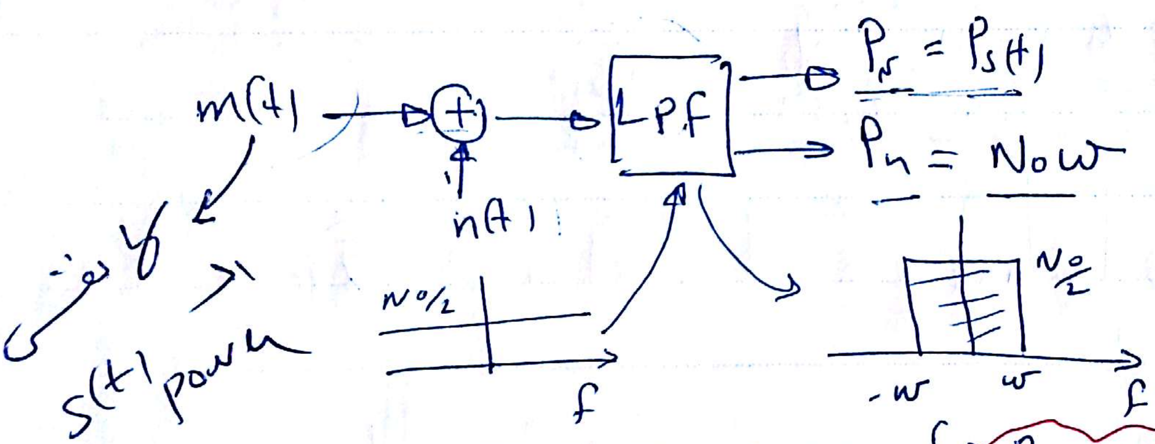
مقدمة

$$\boxed{\text{I} \quad (SNR)_{ch} = (SNR)_{i/p}}$$



$$(SNR)_{ch} = \frac{P_s}{P_n} = \frac{P_{s(t)}}{P_n}$$

ولكن يتم حساب الـ $(SNR)_{ch}$ يتم فرض أنه هناك إشارة $m(t)$ لها
نفس الـ Power الخاصة بـ $s(t)$ ويتم إضافة الـ $n(t)$ عليها كما بالشكل التالي



$$P_s = P_{s(t)}$$

$$P_n = N_o w$$

$$P_n = N_o w$$

$$\therefore (SNR)_{i/p} = (SNR)_{ch} = \frac{P_{s(t)}}{N_o w}$$

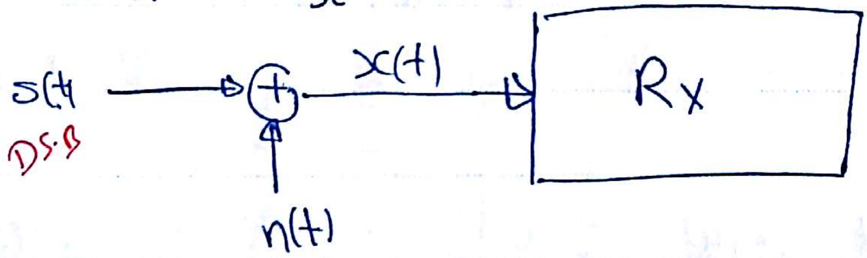
والغرض السابق سوف يتم استخدامه مع جميع أنواع Mod
وسوف نلاحظ أنه الجزء المهم في حسابه هو الـ $P_{s(t)}$ واللي
سوف تختلف من نظام إلى آخر

[I] Noise In D.S.B.S.C

اللهم

ⓐ (SNR) i/p

At (Tx) o/p : $s(t)_{DSBSC} = A_c\, m(t) \cos 2\pi f_c t$

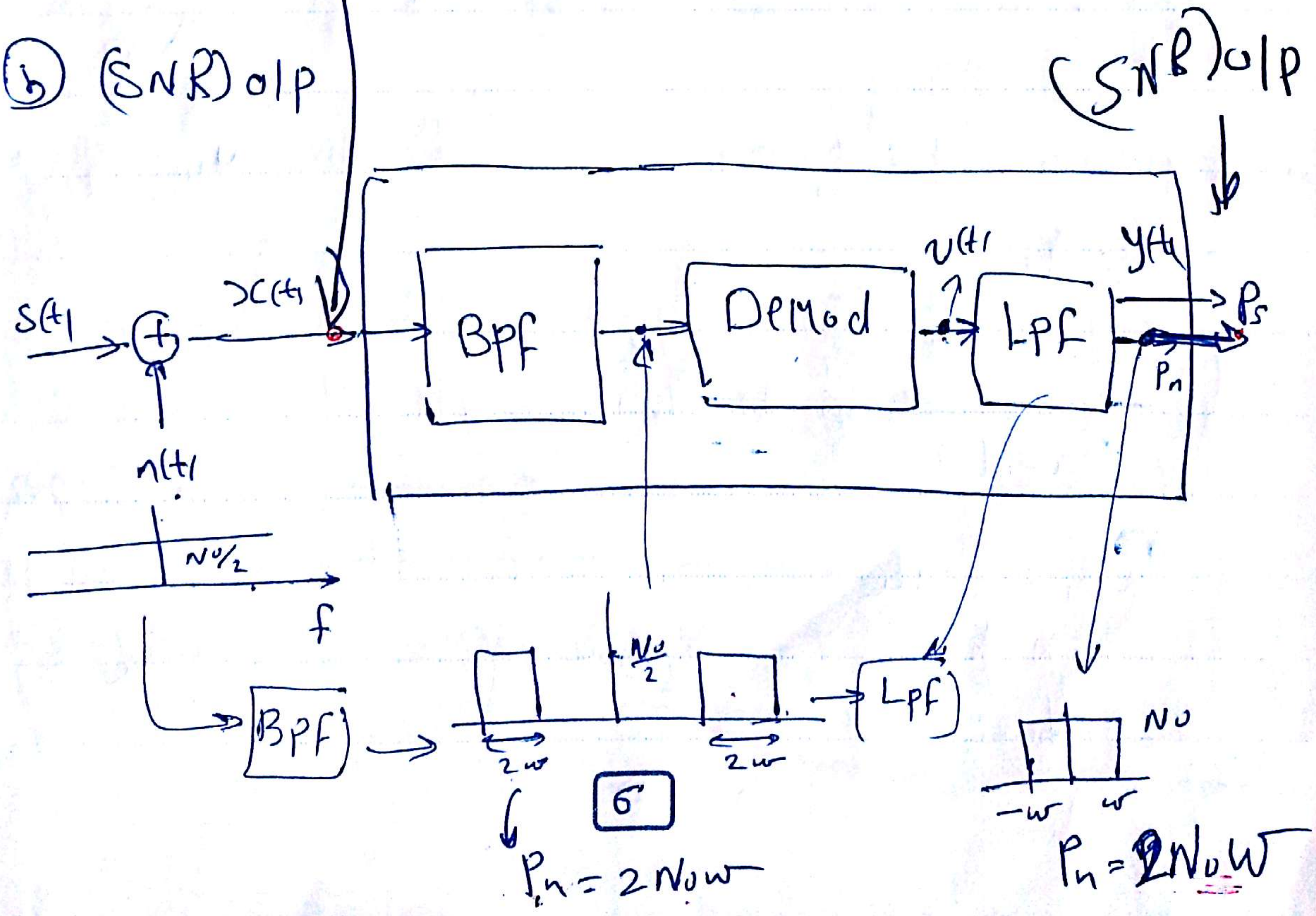


P → power ال

$$x(t) = s(t) + n(t)$$

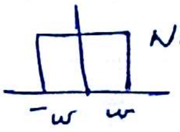
$$\boxed{(SNR)_{i/p} = \frac{P_{s(t)}}{P_n} = \frac{A_c^2 P/2}{N_o W} = \frac{A_c^2 P}{2(N_o W)}} \quad (1)$$

ⓑ (SNR) o/p



6

من الشكل السابق وجدنا أن شكل الـ noise بعد أن تمر على

كلاً من الـ Bpf والـ Lpf تكون N_0  وتكون

الـ $P_{n\,o/p} = 2N_0 w$ ولكن هنا الرقم ليس نهائي لأننا لم

نأخذ في الاعتبار تأثير الـ DeMod ع الخرج والذي سوف

ندرسه بالمعادلات في الشكل التالي

$$\boxed{x(t) = s(t) + n(t)} \rightarrow \text{i/p of Rx}$$

$$= A_c\, m(t) \cos 2\pi f_c t + \left[n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t\right]$$

After DeMod

$$v(t) = x(t) \cdot \cos 2\pi f_c t \quad [\text{Coherent Detection}]$$

$$v(t) = A_c\, m(t) \cos^2 2\pi f_c t + n_I(t) \cos^2 2\pi f_c t - n_Q(t) \sin 2\pi f_c t * \cos 2\pi f_c t$$

$$= \frac{1}{2} A_c\, m(t) \left[1 + \cos 4\pi f_c t\right] + \frac{1}{2} n_I(t) \left[1 + \cos 4\pi f_c t\right] + \frac{1}{2} n_Q(t) \sin 4\pi f_c t$$

After (LPF) around 0

$$\boxed{y(t) = \underbrace{\frac{1}{2} A_c\, m(t)}_{\text{signal}} + \underbrace{\frac{1}{2} n_I(t)}_{\text{noise}}}$$

signal noise

$$y(t) = \underbrace{\tfrac{1}{2} A_c\, m(t)}_{\text{signal}} + \underbrace{\tfrac{1}{2} n_I(t)}_{\text{noise}}$$

. noise

$$\left(P_o = \frac{1}{4} A_c^2\, P\right) + \left(\frac{1}{4}\,(P_n)\right)$$

تم حسابه بمخرج الفلتر $= 2N_o W$

$$\therefore P_o = \frac{1}{4} A_c^2\, P \rightarrow \text{o/p signal}$$

$$P_{n_o} = \left(\frac{1}{4}\right)(2N_o W) = \left(\frac{N_o W}{2}\right) \rightarrow \text{o/p Noise}$$

$$\boxed{\therefore (SNR)_o = \frac{\frac{1}{4} A_c^2\, P}{\frac{N_o W}{2}} = \frac{A_c^2\, P}{2 N_o W}} \rightarrow (2)$$

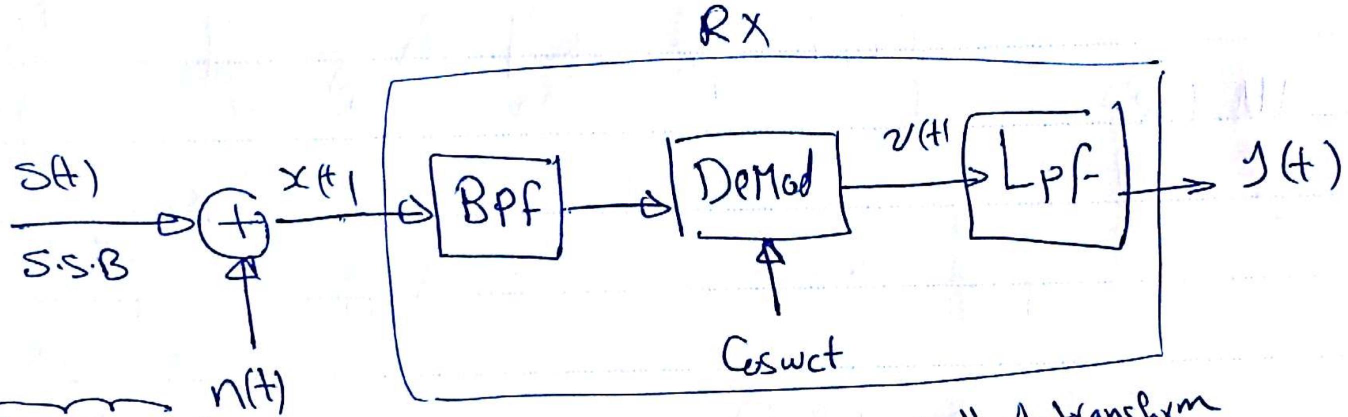
$$\boxed{(SNR)_{ch} = \frac{A_c^2\, P}{2 N_o W}} \rightarrow \text{from } (1)$$

$$\therefore \frac{(SNR)_{o/p}}{(SNR)_{i/p}} = 1 = \text{FOM}$$

وبالتالي ال System لا يضيف ولا يقلل من ال Noise

[2] Noise in S.S.B Modulation

نفس الطريقة مع اختلاف معادلات ال S.S.B



(SNR)ch

helbert transform

$$s(t) = \frac{A_c}{2} m(t) \cos 2\pi f_c t + \frac{A_c}{2} \hat{m}(t) \sin 2\pi f_c t$$

$$P_{s(t)} = \frac{A_c^2}{8} P + \frac{A_c^2 P}{8} = \frac{A_c^2 P}{4}$$

$$P_{n\,i/p} = N_0 w$$

$$\therefore (SNR)_c = \frac{A_c^2 P/4}{N_0 w} = \frac{A_c^2 P}{4(N_0 w)} \quad (1)$$

(SNR) o/p

assume lower side Band

$$x(t) = s(t) + n(t)$$

السنتر

$$x(t) = s(t) + n_I(t) \cos 2\pi\left(f_c - \frac{w}{2}\right)t - n_Q(t) \sin 2\pi\left(f_c - \frac{w}{2}\right)t$$

المركز

after Demodulation

$$v(t) = x(t) \cdot \cos 2\pi f_c t \rightarrow \text{Coherent detection}$$

مفيد

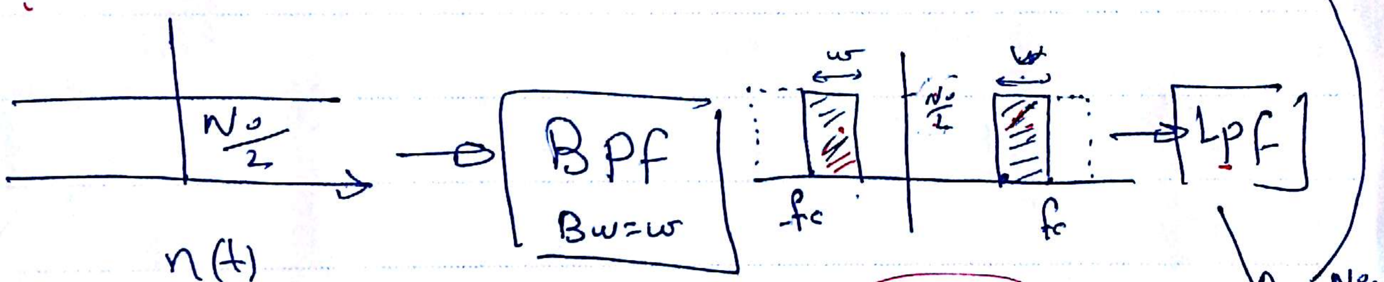
After LPF

متحصل عليه من الفلتر

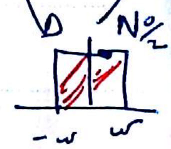
$$y(t) = \underbrace{\frac{A_c}{4} m(t)}_{o/p\ signal} + \underbrace{\frac{1}{2}\overbrace{\left[n_I(t)\cos w_c t - n_Q(t)\sin w_c t\right]}^{n(t)}}_{o/p\ noise}$$

$$\boxed{P_{s\,o/p} = \frac{A_c^2 P}{16}}$$

$$P_{n\,o/p} = \left(\frac{1}{4}\right)(\text{ من الفلتر })$$



$P_n = N_o w$



$P_n = N_o w$

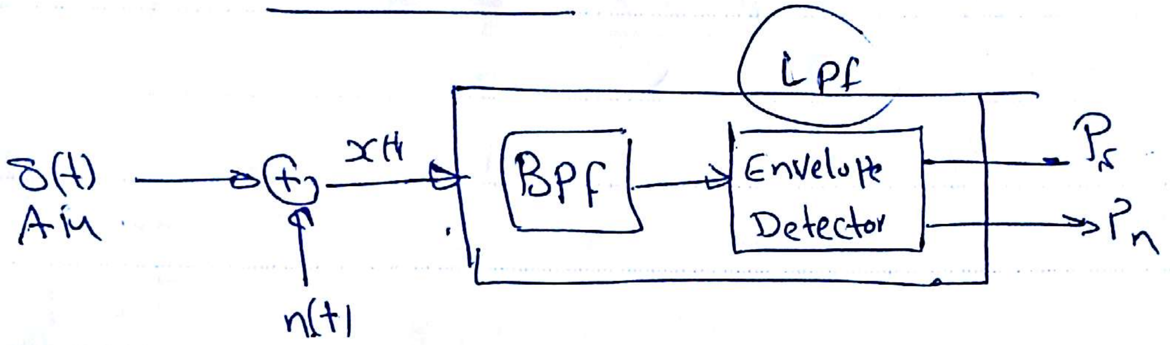
$$\boxed{P_{n\,o/p} = \left(\frac{1}{4}\right) N_o w = \frac{N_o w}{4}}$$

$$\boxed{\therefore (SNR)_{o/p} = \frac{A_c^2 P/16}{N_o w/4} = \frac{A_c^2 P}{4 N_o w}}$$

$$(SNR)_{i/p} = \frac{A_c^2 P}{4 N_o w} = (SNR)_{o/p}$$

$$FoM = 1$$

(3) AM - T.C



① $(SNR)_{i/p}$

$$\boxed{s(t) = A_c[1+k_a m(t)] \cos w_c t}$$

$$= A_c \cos w_c t + A_c k_a m(t) \cos w_c t$$

$$P_{s(t)} = \frac{A_c^2}{2} + \frac{A_c^2 k_a^2 P}{2} = \frac{A_c^2}{2}[1+k_a^2 P]$$

$$P_n = N_o w$$

$$\boxed{\therefore (SNR)_{ch} = \frac{A_c^2[1+k_a^2 P]}{2(N_o w)}} \rightarrow ①$$

③ $(SNR)_{o/p}$

$\because x(t) = s(t) + n(t)$ ① $\quad , \; n(t) = n_I(t) \cos w_c t - n_Q(t) \sin w_c t$

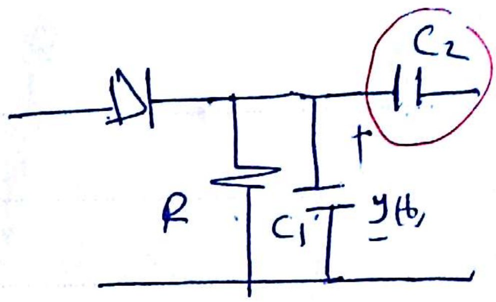
$x(t) = [A_c + A_c k_a m(t) + n_I(t)] \cos w_c t - n_Q(t) \sin w_c t$ ③

$x(t) = (A_c \cos w_c t) + (A_c k_a m(t) \cos w_c t) + (n_I(t) \cos w_c t - n_Q(t) \sin w_c t)$ ②

11

After envelope Detector

$$y(t) \cong \underbrace{A_c}_{DC} + A_c k_a m(t) + n_I(t)$$

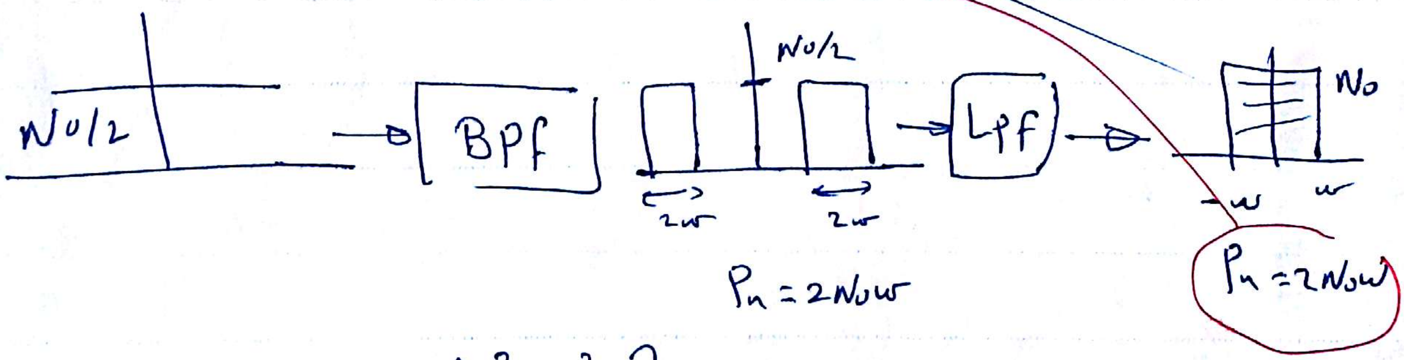


$\boxed{\text{After } C_2}$

$$\boxed{y(t) = \underbrace{A_c k_a m(t)}_{\text{signal}} + \underbrace{(1) n_I(t)}_{\text{noise}}}$$

$$P_{S\,o/p} = A_c^2 k_a^2 P$$

$$\boxed{P_{o/p\,noise} = (1)(2N_0 w) \rightarrow \text{From filter}}$$



$$\therefore (SNR)_{o/p} = \frac{A_c^2 k_a^2 P}{2N_0 w}$$

$$(SNR)_{ch\,i/p} = \frac{A_c^2 [1 + k_a^2 P]}{2N_0 w}$$

$\therefore$ Figure of Merit $= FOM = \dfrac{K_a^2 P}{1+K_a^2 P} < 1$

* نلاحظ ال $(SNR)_{o/p}$ أقل من ال $(SNR)_{i/p}$ وذلك بسبب وجود ال Carrier الذي يستهلك جزء كبير من ال power ولا يحتوي ع ال معلومه .

Example

Single tone Modulation

Consider $\boxed{m(t) = A_m \cos 2\pi f_m t}$

$m(t) \to \cos, \sin$

Find FoM for AM system

(Solution)

$\therefore m(t) = A_m \cos 2\pi f_m t$

$$\boxed{\therefore P = A_m^2/2}$$

$$s(t) = A_c[1+\underbrace{K_a A_m}_{\mu} \cos 2\pi f_m t] \cos 2\pi f_c t$$

$$= A_c[1+\mu \cos 2\pi f_m t] \cos 2\pi f_c t$$

$\boxed{K_a A_m = \mu}$

$$\therefore FoM = \frac{(SNR)_o}{SNR_i} = \frac{K_a^2 P}{1+K_a^2 P} = \frac{K_a^2 A_m^2/2}{1+K_a^2 \frac{A_m^2}{2}} = \frac{\mu^2/2}{1+\frac{\mu^2}{2}}$$

$$FoM = \frac{\mu^2}{2+\mu^2}, \text{ for } \mu = 1 \Rightarrow FoM_{max} = \frac{1}{3}$$

ملخص القوانين السابقة

System	$(SNR)_{i/p}$	$(SNR)_{o/p}$	FOM
DSB S.C	$\frac{A_c^2 P}{2N_0 W}$	$\frac{A_c^2 P}{2N_0 W}$	1
S.S.B	$\frac{A_c^2 P}{4N_0 W}$	$\frac{A_c^2 P}{4N_0 W}$	1
AM	$\frac{A_c^2 [1+k_a^2 P]}{2N_0 W}$	$\frac{A_c^2 k_a^2 P}{2N_0 W}$	$\frac{k_a^2 P}{1+k_a^2 P} < 1$

	o/p noise معامل ال	(filter) P_n	Total $P_{n\,o/p}$
DSB	$\frac{1}{4} = \left(\frac{1}{2}\right)^2$	$2N_0 W$	$\frac{N_0 W}{2}$
S.S.B	$\frac{1}{4}$	$N_0 W$	$\frac{N_0 W}{4}$
AM	1	$2N_0 W$	$2N_0 W$

الفرقة الثانية ـاتصالات مادة : هندسة الاتصالات

Sheet 1 (Noise)

1-a A DSB-SC modulated signal is transmitted over a noisy channel, with the power spectral density of the noise being as shown in Figure P2.46. The message bandwidth is 4 kHz and the carrier frequency is 200 kHz. Assuming that the average power of the modulated wave is 10 watts, determine the output signal-to-noise ratio of the receiver.

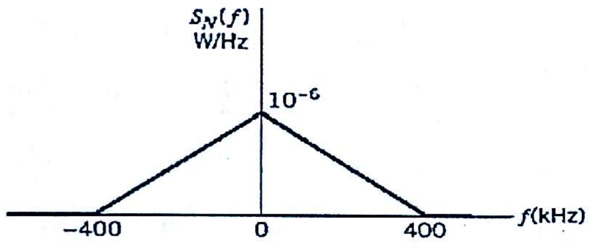


FIGURE P2.46

1-b Repeat for SSB – SC with upper side

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2- A certain communication channel is characterized by 90-dB attenuation and additive white noise with power-spectral density of $\frac{N_0}{2} = 0.5 \times 10^{-14}$ W/Hz. The bandwidth of the message signal is 1.5 MHz and its ~~amplitude is uniformly distributed in the interval~~ [−1, 1]. If we require that the SNR after demodulation be 30 dB, in each of the following cases find the necessary transmitter power.

1. USSB modulation.
2. Conventional AM with a modulation index of 0.5. P =1/3 watt
3. DSB-SC modulation.

=====================================

3- Let a message signal $m(t)$ be transmitted using single-sideband modulation. The power spectral density of $m(t)$ is

$$S_M(f) = \begin{cases} a\dfrac{|f|}{W}, & |f| \leq W \\ 0, & \text{otherwise} \end{cases}$$

where a and W are constants. White Gaussian noise of zero mean and power spectral density $N_0/2$ is added to the SSB modulated wave at the receiver input. Find an expression for the output signal-to-noise ratio of the receiver.

4- Report

In a broadcasting communication system the transmitter power is 40 KW, the channel attenuation is 80 dB, and the noise power-spectral density is 10^{-10} W/Hz. The message signal has a bandwidth of 10^4 Hz.

a- Find the output SNR if the modulation is DSB.

b- Find the output SNR if the modulation is SSB.

c- Find the output SNR if the modulation is conventional AM with a modulation index of 0.85 and normalized message power of 0.2.

sheet ①

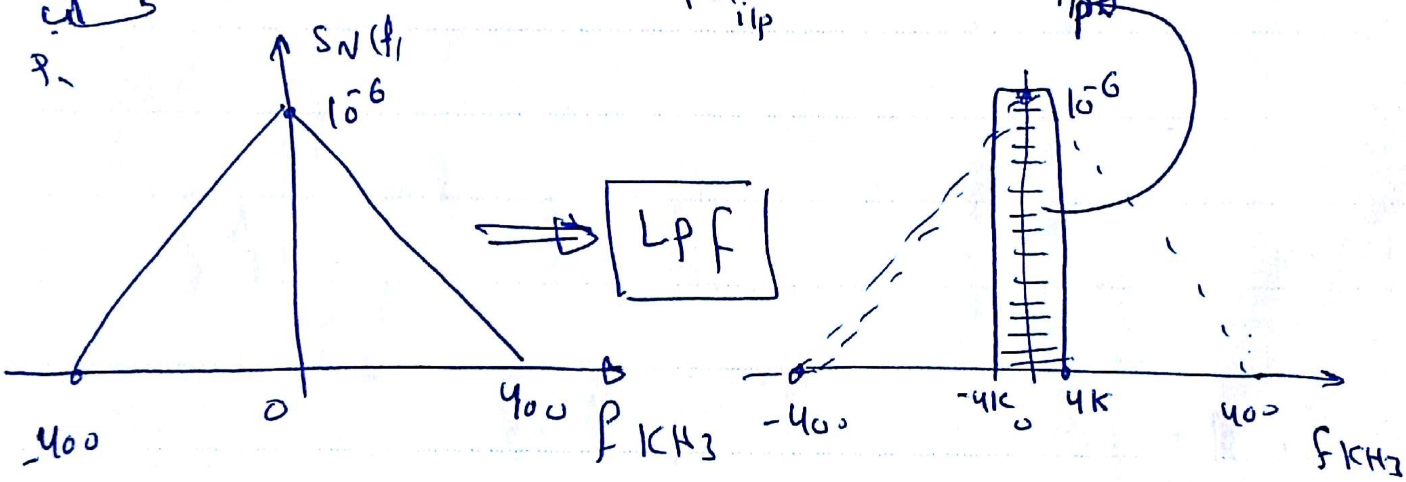
① ⓐ $W = 4\,KHz \quad f_c = 200\,KHz \quad P_{s(t)} = 10\,watts$

$(SNR)_{o/p} = ??$

ⓐ for D.S.B.SC SOL

$\because (SNR)_{i/p} = (SNR)_{o/p}$

$\because (SNR)_{i/p} = \dfrac{P_{s(t)}}{P_{n_{i/p}}} = \dfrac{10\,watt}{P_{n_{i/p}}}$



$\therefore P_n = 10^{-6} \times 8\,KHz = 8 \times 10^{-3}\,watt$

$$\boxed{\therefore (SNR)_{i/p} = \frac{10}{8 \times 10^{-3}} = 1250 = SNR_o} \quad \#$$

ⓑ $\boxed{\text{For SS.B}}$

$(SNR)_{i/p} = (SNR)_{o/p} = \dfrac{P_{s(t)}}{P_n}$

$\because P_{s(t)} = 10\,watts$

$P_{n_{SS.B\ i/p}} = P_{n_{D.S.B\ i/p}} = 10^{-6} \times 8K = 8 \times 10^{-3}\,watt \quad \boxed{1}$

$\therefore (SNR)_{o/p} = \dfrac{10}{8 \times 10^{-3}} = 1250 \quad \#$

$\boxed{2}$ Attenuation = 90 dB

$\frac{N_o}{2} = 0.5 \times 10^{-14}$ W/Hz $\rightarrow N_o = 10^{-14}$ W/Hz

$W = 1.5$ MHz

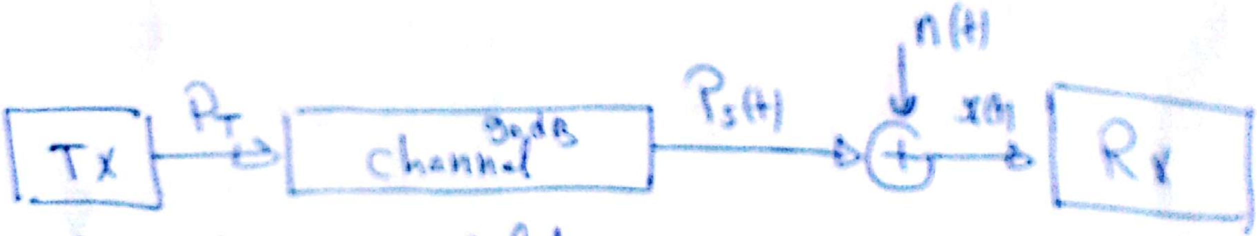
$(SNR)_{o/p} = 30\ dB = 1000$

$P_T = ??$

Sol

Channel attenuation $= 10 \log \frac{P_T}{P_{s(t)}} = 90\ dB$

$\therefore \boxed{P_{s(t)} = 10^{-9} P_T}$



ⓐ for USSB Modulation :-

$(SNR)_{o/p} = (SNR)_{i/p} = 1000 \rightarrow$ required

$\therefore (SNR)_{i/p} = \frac{P_{s(t)}}{P_{n\,i/p}} = \frac{10^{-9} P_T}{N_o W} = 1000$

$\therefore \frac{10^{-9} P_T}{10^{-14} \times 1.5 \times 10^{6}} = 1000 \rightarrow \boxed{P_T = 15 KW}$ ##

$\boxed{2}$ $\boxed{2}$

(b) For D.S.B.SC

$$(SNR)_{i/p} = (SNR)_{o/p} = \frac{P_{s(t)\, i/p}}{P_{n\, i/p}} = \frac{10^{-9} P_T}{N_0 \omega} = 1000$$

$$\therefore \boxed{P_{T_{D.S.B.SC}} = 15\, Kw}$$

(c) For AM with $\overline{P} = 1/3$ watt and $K_a = 0.5$

$$\because \frac{(SNR)_{o/p}}{(SNR)_{i/p}} = \frac{K_a^2 \overline{P}}{1 + K_a^2 \overline{P}}$$

$$\therefore (SNR)_{o/p} = \frac{K_a^2 P}{1 + K_a^2 P} (SNR)_{i/p}$$

$$\therefore 1000 = \frac{(0.5)^2 \frac{1}{3}}{1 + (0.5)^2 \frac{1}{3}} * \frac{P_{s(t)}}{N_0 \omega}$$

$$1000 = \frac{\frac{0.25}{3}}{1 + \frac{0.25}{3}} * \frac{10^{-9} P_T}{10^{-4} * 1.5 * 10^{6}}$$

$$\boxed{P_T = 195\, Kw}$$

$\frac{\boxed{3}}{\boxed{3}}$

$\boxed{3}$ for S.S.B

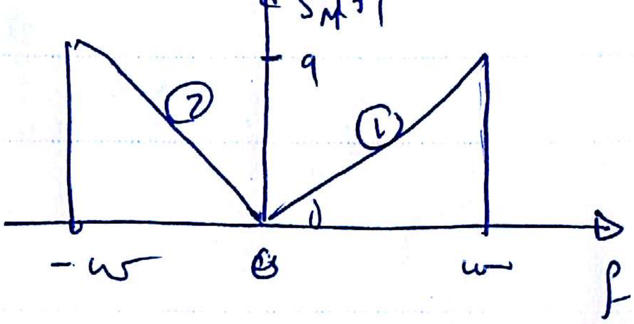
$$S_m(f) = \begin{cases} a\frac{|f|}{w} & |f| \leq w \\ 0 & o.w \end{cases}$$

$(SNR)\ o/p = ??$

SoL

$$\because (SNR)_{o/p\ S.S.B} = \frac{A_c^2 P}{4N_0 w} \quad \leftarrow P \text{ is } m(t) \text{ Power}$$

$$\leftarrow \because P = \int_{-w}^{w} S_M(f)\, df$$



$$P = 2\int_0^w \frac{a}{w} f \cdot df = \frac{2a}{w}\left[f^2/2\right]_0^w = \frac{a}{w} w^2 = aw$$

$$\boxed{P = aw}$$

$$\boxed{\therefore (SNR)_{o/p} = \frac{A_c^2 a \cancel{w}}{4N_0 \cancel{w}} = \frac{A_c^2 a}{4N_0}}$$

Note

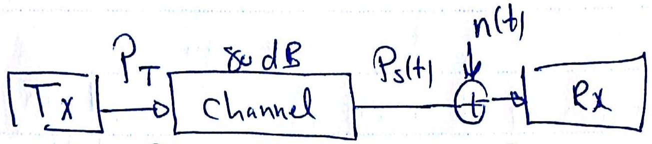
$$S_M(f) = \begin{cases} a\frac{|f|}{w} & |f| \leq w \\ 0 & o.w \end{cases} = \begin{cases} \frac{af}{w} & f \leq w \quad (1) \\ -\frac{af}{w} & -f \leq w = f \geq -w \quad (2) \\ 0 & \text{otherwise} \end{cases}$$

(4) $P_T = 40\,kW$ Channel attenuation $= 80\,dB$

$\frac{N_o}{2} = 10^{-10}$ $W = 10^4\,Hz$

Sol

(a) for D.S.B



$\because\ 80\,dB = 10\log\frac{P_T}{P_{s(t)}} = 10\log\frac{40*10^3}{P_{s(t)}}$

$\therefore\ \boxed{P_{s(t)} = 4*10^{-4}\ \text{watt}}$

$\therefore (SNR)_{i/p\ DSB\cdot SC} = \frac{P_{s(t)}}{N_o W} = \frac{4*10^{-4}}{2*10^{-10}*10^4} = 2*10^2$

$\boxed{\therefore (SNR)_{o/p} = (SNR)_{i/p} = 2*10^2}$

(b) for S.S.B

$(SNR)_{i/p} = (SNR)_{o/p} = \frac{P_{s(t)}}{N_o W} = \frac{4*10^{-4}}{2*10^{-10}*10^4}$

$(SNR)_{o/p} = 2*10^2$ #

(c) <u>For AM</u> $K_a = 0.8K$ & $P = 0.2$ watt

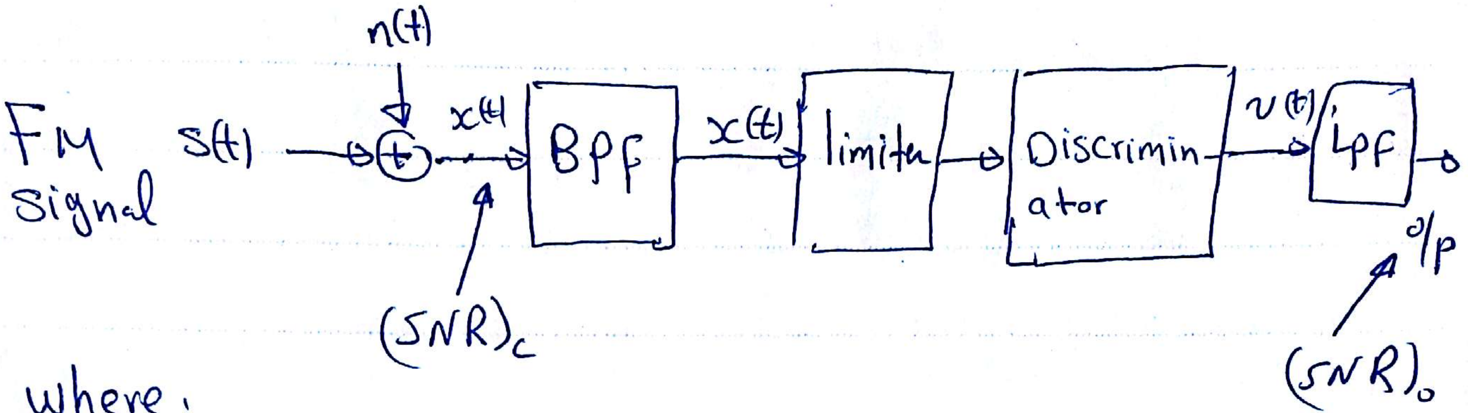
$$SNR(i/p) = \frac{P_{s(t)}}{N_o w} = 2 \times 10^2$$

<u>But</u> $$(SNR)_o = \frac{K_a^2 P}{1 + K_a^2 P} (SNR)_{i/p}$$

$$(SNR)_o = \frac{(0.85)^2 \times 0.2}{1 + (0.85)^2 \times 0.2} \times 2 \times 10^2$$

$$\boxed{(SNR)_{o\,AM} = 0.113 \times 2 \times 10^2}$$

Noise In FM Receiver



where :-

$$s(t)_{FM} = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t) \cdot dt\right]$$

$$\therefore P_{s(t)} = \frac{A_c^2}{2} \quad \& \quad P_{n_{i/p}} = N_o w$$

$$\boxed{\therefore (SNR)_c = \frac{A_c^2}{2N_o w}} \quad (1)$$

⇒ $(SNR)_{o/p}$

let $n(t) = n_I(t) \cos w_c t - n_Q(t) \sin w_c t$

$$\therefore n(t) = r(t) \cos[(2\pi f_c t) + \psi(t)]$$

$$\& \text{ let } \phi(t) = 2\pi k_f \int_0^t m(t) \cdot dt$$

$$\therefore x(t) = s(t) + n(t)$$

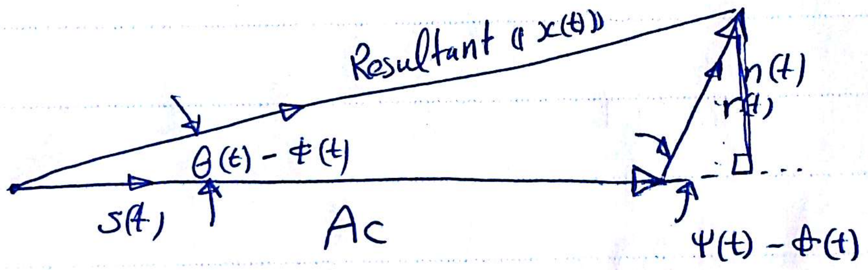
$$x(t) = A_c \cos[(2\pi f_c t) + \phi(t)] + r(t) \cos[2\pi f_c t + \psi(t)]$$

let $x(t)$ has phase of $\theta(t)$ → الحصلة

$s(t)$ has phase $= \phi(t)$

$r(t)$ has phase $= \psi(t)$

والآن سوف تم حساب ال $\theta(t)$ بمعلومية ال $\phi(t)$ و ال $\psi(t)$



From the phasor diagrame.

$$\theta(t) - \phi(t) = \tan^{-1} \frac{\text{المقابل}}{\text{المجاور}} \Rightarrow \theta(t) = \phi(t) + \tan^{-1} \frac{\text{المقابل}}{\text{المجاور}}$$

$$\therefore \theta(t) = \phi(t) + \tan^{-1}\left[\frac{r(t) \sin[\psi(t) - \phi(t)]}{A_c + r(t) \cos[\psi(t) - \phi(t)]}\right]$$

$\because \tan^{-1}\alpha = \alpha$ when α is very small

$$\therefore \theta(t) = \phi(t) + \frac{r(t) \sin(\psi(t) - \phi(t))}{\underset{\text{كبير}}{A_c} + \underbrace{r(t) \cos[\psi(t) - \phi(t)]}_{\text{صغير بالنسبة لل } A_c}}$$

$$\therefore \theta(t) \cong \phi(t) + \frac{r(t)\sin[\psi(t) - \phi(t)]}{A_c}$$

$$\therefore \theta(t) \cong \phi(t) + \frac{r(t)}{A_c}\sin[\psi(t) - \phi(t)]$$

$$\theta(t) \cong 2\pi k_f \int_0^t m(t)\cdot dt + \frac{r(t)}{A_c}\sin[\psi(t) - \phi(t)]$$

$\Rightarrow$ At Discriminator o/p :-

$$v(t) = \frac{1}{2\pi}\frac{d\theta(t)}{dt} =$$

$$= \frac{1}{\cancel{2\pi}}\frac{d}{dt}\left[\cancel{2\pi} k_f \int_0^t m(t)\cdot dt + \frac{r(t)}{A_c}\sin[\psi(t) - \phi(t)]\right]$$

$$\boxed{\therefore v(t) = \underbrace{k_f\, m(t)}_{\text{Signal}} + \underbrace{n_d(t)}_{\text{noise}}}$$

$$\boxed{n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}[r(t)\sin(\psi(t) - \phi(t))]}$$

$$\boxed{\therefore P_o = k_f^2\, P} \Longrightarrow \text{I}$$

فاضل حساب ال P_n مع المعادلة $n_d(t)$

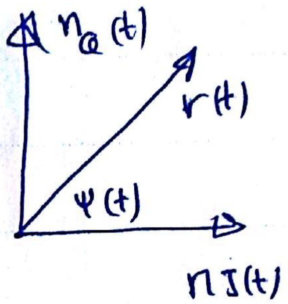
$$n_d(t) = \frac{1}{2\pi A_c}\frac{d}{dt}[r(t)\sin(\psi(t) - \phi(t))]$$

$$n_d(t) \cong \frac{1}{2\pi A_c}\frac{d}{dt}[r(t)\sin\psi(t)]$$

نهمل

في ال noise

$$\therefore n_d(t) \simeq \frac{1}{2\pi A_c}\frac{d}{dt}\left[r(t)\sin\psi(t)\right]$$



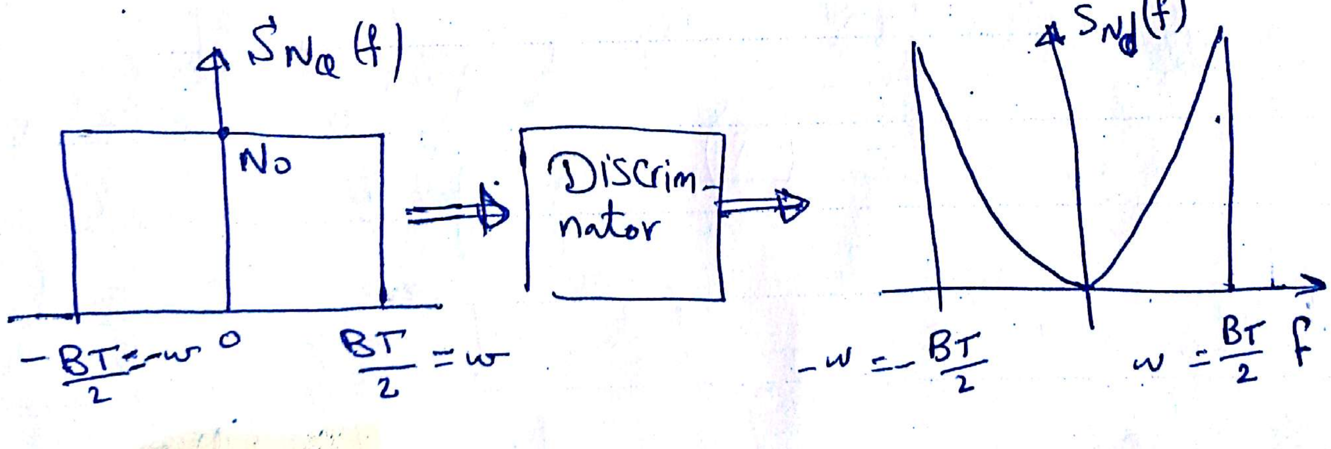
But $r(t)\sin\psi(t) = n_Q(t)$

$$\boxed{\therefore n_d(t) \simeq \frac{1}{2\pi A_c}\frac{dn_Q(t)}{dt}}$$

وإذا سوف نعمل ع الـ freq Domain بدلاً من الـ time Domain وسوف
ليتقبل الـ التفاضل بـ J2πf وسوف نربع واحسب الـ Power

$$\therefore |S_{N_d}(f)| = \left|\frac{(J\cancel{2\pi}f)^2}{(\cancel{2\pi}A_c)^2}S_{N_Q}(f)\right| = \frac{f^2}{A_c^2}S_{N_Q}(f)$$

$$\boxed{\therefore S_{N_d}(f) = \frac{f^2}{A_c^2}S_{N_Q}(f)} \rightarrow \text{III}$$



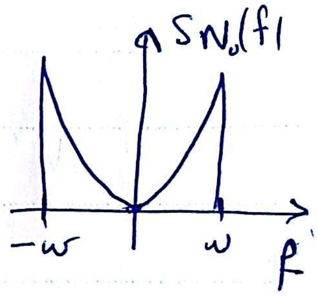
where:

$$S_{N_Q}(f) = \begin{cases} N_o & |f| \leq \frac{B_T}{2} \\ 0 & \text{otherwise} \end{cases}$$

$$S_{Nd}(f) = \frac{f^2}{A_c^2} S'_{N_Q}(f) = \begin{cases} \frac{N_o f^2}{A_c^2} & |f| \leq \frac{B_T}{2} \\ 0 & \text{otherwise} \end{cases}$$

After LPF with Bandwidth $= w$

$$\therefore S_{N_o}(f) = \begin{cases} \frac{N_o f^2}{A_c^2} & |f| \leq w \\ 0 & \text{otherwise} \end{cases}$$



وحساب ال $P_{n\,o/p}$ تم التكامل من $(-w \to w)$ لـ $S_{N_o}(f)$

$$\boxed{\therefore P_{n\,o/p} = \int_{-w}^{w} \frac{N_o f^2}{A_c^2} \cdot df = \frac{2 N_o w^3}{3 A_c^2}} \quad \#$$

$\because P_s = P_o = K_f^2 P$ from (I)

$$\boxed{\therefore (SNR)_{o/p} = \frac{K_f^2 P}{\frac{2 N_o w^3}{3 A_c^2}} = \frac{3 A_c^2 K_f^2 P}{2 N_o w^3}} \quad \#$$

$$\boxed{\therefore (SNR)_{i/p} = \frac{A_c^2}{2N_0W}} \rightarrow$$

$$\therefore FOM = \frac{3\cancel{A_c^2}k_f^2P}{\cancel{2N_0}W^3} \div \frac{\cancel{A_c^2}}{\cancel{2N_0}W}$$

$$\boxed{FOM = \frac{3k_f^2P}{W^2}} \quad \#$$

Ex Single tone Modulation

where

$$m(t) = A_m \cos 2\pi f_m t$$

$$\therefore \boxed{\Delta f = k_f A_m} \rightarrow \text{peak frequency deviation}$$

$$\therefore S_{FM}(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t).dt\right]$$

$$= A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t A_m \cos(2\pi f_m t).dt\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{\cancel{2\pi} k_f A_m}{\cancel{2\pi} f_m} \sin 2\pi f_m t\right]$$

$$= A_c \cos\left[2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right]$$

$$s(t) = A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t\right]$$

$$\because m(t) = A_m \cos 2\pi f_m t = \frac{\Delta f}{K_f} \cos(2\pi f_m t)$$

$$\boxed{\therefore P = \frac{\Delta f^2}{2K_f^2}} \rightarrow (1)$$

$$\therefore (SNR)_o = \frac{3A_c^2 K_f^2 P}{2N_o w^3} = \frac{3A_c^2 \cancel{K_f^2}\, \Delta f^2 / 2\cancel{K_f^2}}{2N_o w^3}$$

$$= \frac{3A_c^2 \Delta f^2}{4N_o w^3} = \frac{3A_c^2 \beta^2}{4N_o w} \quad \because \beta = \frac{\Delta f}{w}$$

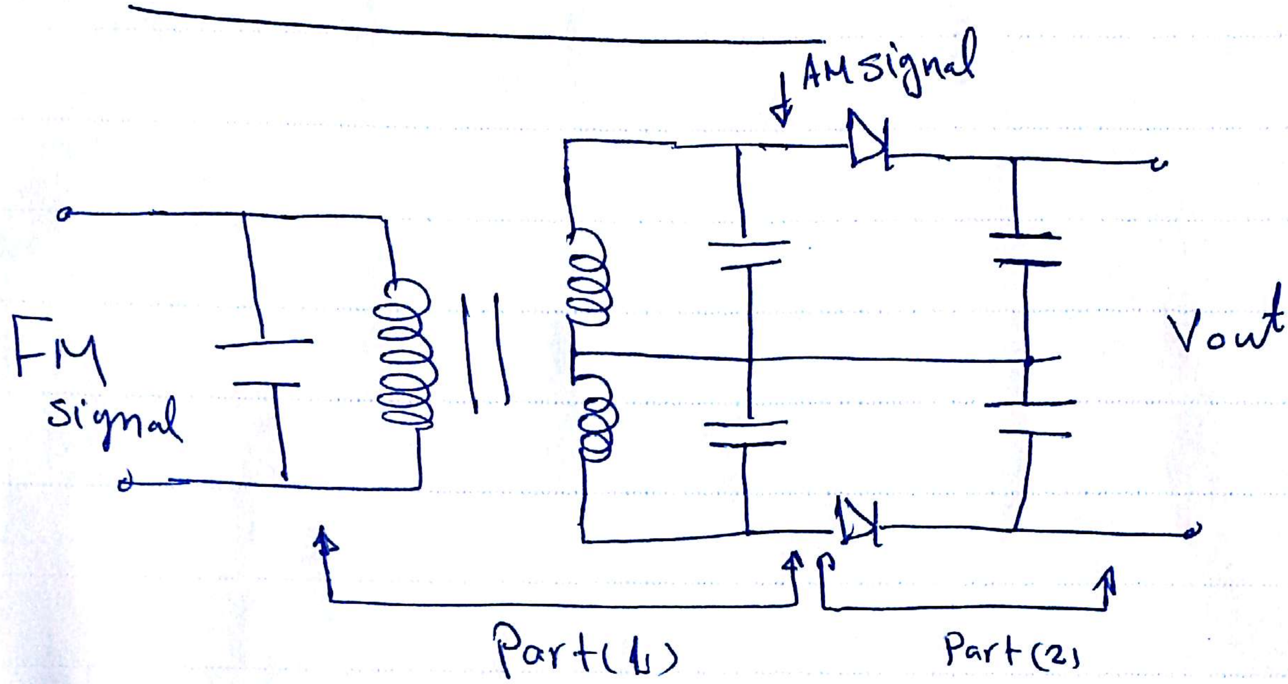
$$\because \beta = \frac{\Delta f}{w} = \text{Modulation index}$$

$$(SNR)_{ch} = \frac{A_c^2}{2N_o w}$$

$$\therefore FOM = \frac{(SNR)_o}{(SNR)_c} = \frac{3}{2}\left(\frac{\Delta f}{W}\right)^2 = \frac{3}{2}\beta^2$$

single tone

Discriminator Circuit



وهي دائرة تتكون من جزئين

① tunned circuit

* وهي المسئولة عن عملية التفاضل للإستخلاص m(t) من داخل ال Carrier وهي عبارة عن two tunned circuits

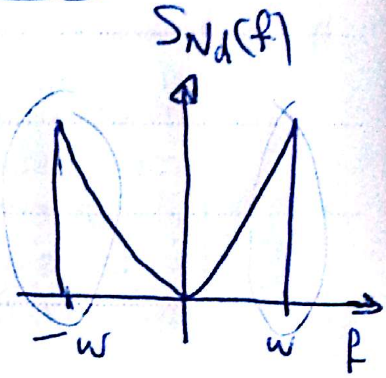
كما أنها تقوم بتحويل الإشارة من FM إلى AM

② Envelope detector

* وهو المسئول عن إستخلاص الإشارة m(t) من ال AM

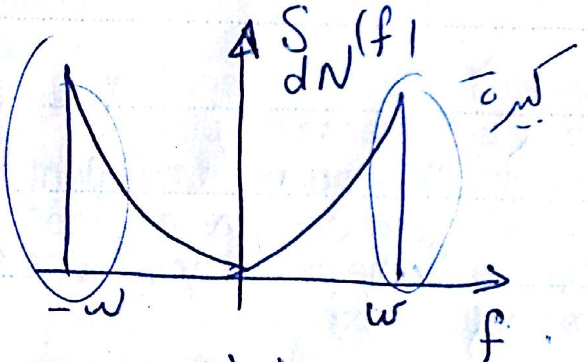
PRE-Emphasis and DE-Emphasis In FM

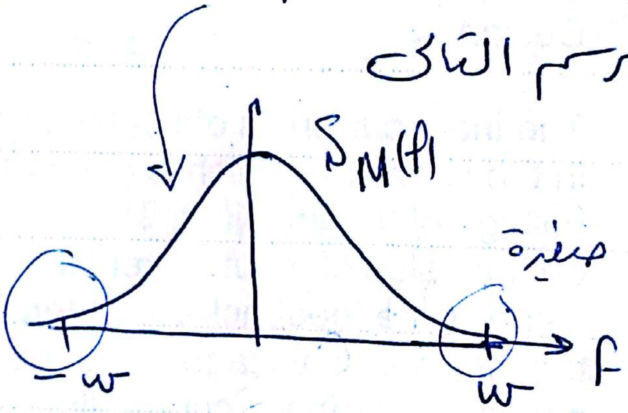
$$* \ \therefore \ S_{Nd}(f) = \begin{cases} \dfrac{N_o f^2}{A_c^2} & |f| < W \\ 0 & \text{otherwise} \end{cases}$$



⇐ نلاحظ انه ال noise power مركز بقيمة كبيرة عند $(W,-W)$

وبالمقارنه بـ توزيع الـ power الخاصه بـ ال m(t) نلاحظ
انه الـ m(t) power صغير جداً عند $-W<W$ كما
بالرسم التالي



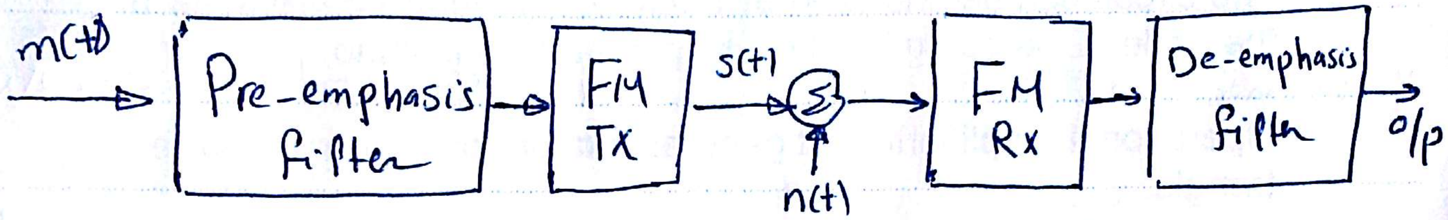


وبالتالي فإنه خرج ال FM Rx سوف يتأثر بشكل كبير
بـ noise عند الترددات القريبه من $-W<W$ وسوف
يحدث تشوه للخرج .

⇐ كل تلك المشاكل تم استخدام فلتر في الدخل عند ال TX
يغير توزيع الـ power الخاصه بـ m(t) على الترددات
مع المحافظة على نفس كمية ال power ليصبح

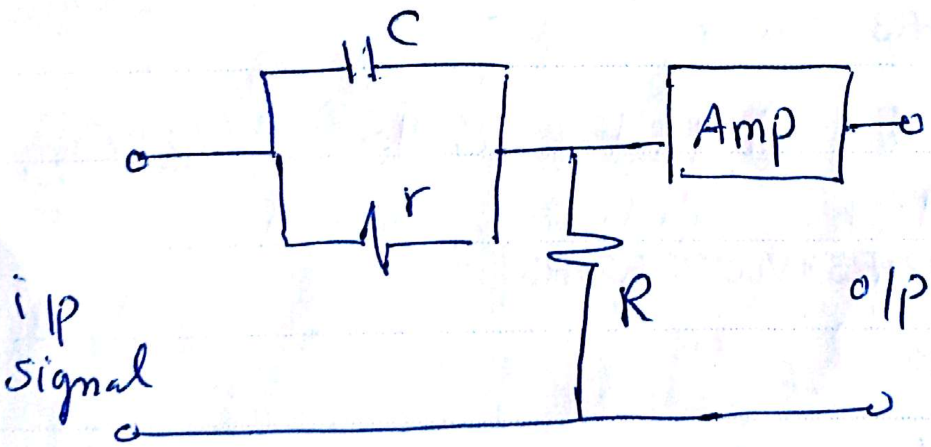
PRE-Emphasis filter.

ويتم إستخدام فلتر أخر في الخرج عند الـ Rx يقوم بالعمليه العكسيه كي تعود الـ m(t) إلى أصلها وتسمى DE-Emphasis filter as Shown in the figure بالاضافة إلى أنه سوف يقلل من قيمة الـ o/p noise

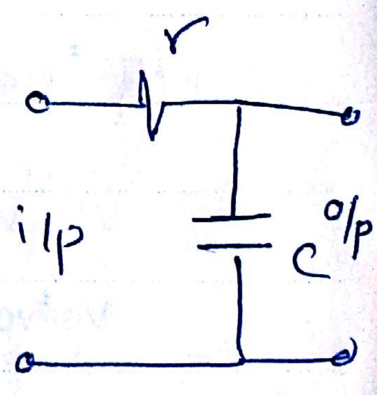


* Pre-emphasis act as high Pass filter
* De-emphasis act as Low pass filter

* $H_{pe}(f) = \dfrac{1}{H_{de}(f)}$ ← H(f) is the transfer function



Pre-emphasis Circuit



Deemphasis Circuit

* Mean o/p noise after De-emphasis filter

$$\int_{-W}^{W} \frac{N_o f^2}{A_c^2}\, df \longrightarrow \boxed{\text{De-emphasis filter}} \longrightarrow \frac{N_o}{A_c^2}\int_{-W}^{W} f^2 \left|H_{De}(f)\right|^2 df$$

$$\boxed{\therefore\ P_{n\,o/p}\ \text{بعد الفلتر} = \frac{N_o}{A_c^2}\int_{-W}^{W} f^2 \left|H_{De}(f)\right|^2 df}$$

احفظ

$$\boxed{P_{n\,o/p\ \text{without De-emphasis}} = \int_{-W}^{W} \frac{N_o f^2}{A_c^2}\, df = \frac{2 N_o W^3}{3 A_c^2}}$$

→ مهم
المحاضرة
السابقة

* D: is the improvement factor

$$I = D = \frac{\text{Average noise Power without DE-Emphasis filter}}{\text{〃 〃 〃 with 〃 〃}}$$

$$D = I = \frac{2 \cancel{N_o} W^3 / 3 \cancel{A_c^2}}{\frac{\cancel{N_o}}{\cancel{A_c^2}}\int_{-W}^{W} f^2 \left|H_{De}(f)\right|^2 df}$$

$$D = I = \frac{2 W^3}{3 \int_{-W}^{W} f^2 \left|H_{de}(f)\right|^2 df}$$

احفظ

$$\boxed{D = I = \frac{2W^3}{3\int_{-W}^{W} f^2 |H_{de}(f)|^2 df}}$$

Ex for FM system where

$$H_{pre}(f) = 1 + \frac{Jf}{f_o} \quad \leftarrow \quad H_{de}(f) = \frac{1}{1 + \frac{Jf}{f_o}}$$

where $f_o = 2.1 \, KHz$

$W = 15 \, KHz$

find D

Sol

$$\therefore H_{de}(f) = \frac{1}{1 + \frac{Jf}{f_o}} \Rightarrow |H_{de}(f)| = \frac{1}{\sqrt{1 + \left(\frac{f}{f_o}\right)^2}}$$

$$\boxed{\therefore |H_{de}(f)|^2 = \frac{1}{1 + \left(\frac{f}{f_o}\right)^2}} \quad (1)$$

$$\therefore D = \frac{2W^3}{3\int_{-W}^{W} f^2 \frac{1}{1 + \left(\frac{f}{f_o}\right)^2} \cdot df}$$

$$D = \frac{2w^3}{3\int_{-w}^{w} f_0^2 \, \frac{f^2/f_0^2 - 1 + 1}{1+\left(\frac{f}{f_0}\right)^2}\, df}$$

$$= \frac{2w^3}{3\int_{-w}^{w} f_0^2 \, \frac{f^2/f_0^2 + 1 - 1}{1+\frac{f^2}{f_0^2}}\, df}$$

$$= \frac{2w^3}{3f_0^2 \int_{-w}^{w} \left(1 - \frac{1}{1+\left(\frac{f}{f_0}\right)^2}\right) df}$$

$$= \frac{2w^3}{3f_0^2 \left[2w - 2f_0 \tan^{-1}\left(\frac{w}{f_0}\right)\right]}$$

$$= \frac{\cancel{2}w^3}{3f_0^{3}\,\cancel{2}\left[\frac{w}{f_0} - \tan^{-1}\left(\frac{w}{f_0}\right)\right]}$$

$$= \frac{(w/f_0)^3}{3\left[\frac{w}{f_0} - \tan^{-1}\left(\frac{w}{f_0}\right)\right]}$$

$$D = \frac{(15/2.1)^3}{3\left[\frac{15}{2.1} - \tan^{-1}\left(\frac{15}{2.1}\right)\right]} = 21$$

Radians

$$\boxed{\therefore D = 10\log(21) = 13\ dB}$$

<u>Ex</u> sheet (2)

(2) sheet (2)

(2.) Suppose that the transfer functions of the pre-emphasis and de-emphasis filters of an FM system are scaled as follows:

$$H_{pe}(f) = k\left(1 + \frac{jf}{f_0}\right)$$

and

$$H_{de}(f) = \frac{1}{k}\left(\frac{1}{1 + jf/f_0}\right)$$

The scaling factor k is to be chosen so that the average power of the emphasized message signal is the same as that of the original message signal $m(t)$.

(a) Find the value of k that satisfies this requirement for the case when the power spectral density of the message signal $m(t)$ is

$$S_M(f) = \begin{cases} \dfrac{S_0}{1 + (f/f_0)^2}, & -W \le f \le W \\ 0, & \text{elsewhere} \end{cases}$$

(b) What is the corresponding value of the improvement factor I produced by using this pair of pre-emphasis and de-emphasis filters?

SoL

$\therefore$ Power of m(t) without pre-emphasis filter = Power of m(t) with filter

$$\therefore \int_{-w}^{w} S_M(f) \cdot df = \int_{-w}^{w} S_M(f) \cdot \left| H_{pre}(f) \right|^2 \cdot df$$

$$\int_{-w}^{w} S_M(f) \cancel{df} = \int_{-w}^{w} \cancel{S_M(f)} \cdot k^2 \left[1 + \left(\frac{f}{f_o} \right)^2 \right] df$$

$$\int_{-w}^{w} \frac{df}{1+\left(\frac{f}{f_o}\right)^2} = \int_{-w}^{w} k^2 df = k^2 2w$$

$$\downarrow$$

$$f_o \tan^{-1} \frac{f}{f_o} \Big|_{-w}^{w} = 2k^2 w$$

$$\cancel{2} f_o \tan^{-1}\left(\frac{w}{f_o}\right) = \cancel{2} w k^2$$

$$\boxed{K = \sqrt{\frac{f_o}{w} \tan^{-1}\left(\frac{w}{f_o}\right)}} \Longrightarrow (a)$$

(b) $\because D = \dfrac{2W^3}{3\int_{-W}^{W} f^2 |H_{de}(f)|^2 df}$

$$D = \frac{2W^3}{3\int_{-W}^{W} \frac{f^2}{K^2} \frac{1}{1+\left(\frac{f}{f_0}\right)^2} \cdot df} \qquad \leftarrow |H_{de}(f)|^2 = \frac{1}{1+\left(\frac{f}{f_0}\right)^2}$$

$$D = \frac{2W^3}{\frac{3f_0^2}{K^2}\int_{-W}^{W} \frac{f^2/f_0^2}{1+\left(\frac{f}{f_0}\right)^2} df}$$

$$D = \frac{2W^3K^2}{3f_0^2\int_{-W}^{W} \frac{1+(f/f_0)^2 - 1}{1+(f/f_0)^2} df}$$

$$= \frac{2W^3K^2}{3f_0^2\int_{-W}^{W}\left[1 - \frac{1}{1+(f/f_0)^2}\right] df}$$

$$D = \frac{\cancel{2} w^3 k^2}{3 f_0^2 \left[\cancel{2} w - \cancel{2} f_0 \tan^{-1}\left(\frac{w}{f_0}\right) \right]}$$

$$D = \frac{\left(w^3 / f_0^3\right) k^2}{3 \left[\frac{w}{f_0} - \tan^{-1}\left(\frac{w}{f_0}\right) \right]}$$

بالتعويض عن قيمة K من (a)

$$D = \frac{\left[w^{\cancel{3}\,2} / f_0^{\cancel{3}\,2} \right] \left[\frac{f_0}{w} \tan^{-1}\left(\frac{w}{f_0}\right) \right]}{3 \left[\frac{w}{f_0} - \tan^{-1}\left(\frac{w}{f_0}\right) \right]}$$

$$D = \frac{\left(\frac{w}{f_0}\right)^2 \tan^{-1}\left(\frac{w}{f_0}\right)}{3 \left[\frac{w}{f_0} - \tan^{-1}\left(\frac{w}{f_0}\right) \right]}$$

sheet (2)

1 Consider a phase modulation (PM) system, with the modulated wave defined by

$$s(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

where k_p is a constant and $m(t)$ is the message signal. The additive noise $n(t)$ at the phase detector input is

$$n(t) = n_I(t)\cos(2\pi f_c t) - n_Q(t)\sin(2\pi f_c t)$$

Assuming that the carrier-to-noise ratio at the detector input is high compared with unity, determine (a) the output signal-to-noise ratio and (b) the figure of merit of the system. Compare your results with the FM system for the case of sinusoidal modulation.

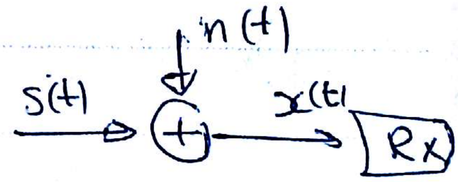
Sol

For PM system

$$s_{PM}(t) = A_c \cos[2\pi f_c t + k_p m(t)]$$

$$\therefore P_{s(t)} = A_c^2/2 \qquad , P_n = N_0 W$$

$$\therefore (SNR)_{ch} = \frac{A_c^2}{2 N_0 W} \rightarrow (1)$$



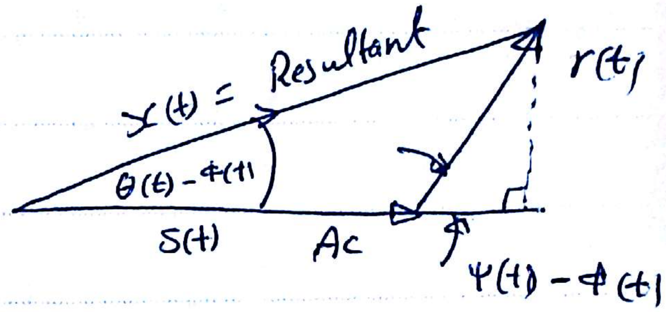
let $\phi(t) = k_p m(t)$

let $n(t) = r(t)\cos[2\pi f_c t + \psi(t)]$

$$\therefore x(t) = s(t) + n(t)$$

$$x(t) = A_c \cos(2\pi f_c t + \phi(t)) + r(t)\cos[2\pi f_c t + \psi(t)]$$

Using phasor diagrame



$$\therefore\ \theta(t) = \phi(t) + \tan^{-1} \frac{r(t)\sin(\psi(t)-\phi(t))}{A_c + \underbrace{r(t)\cos(\psi(t)-\phi(t))}_{\text{نهمل}}}$$

$$\theta(t) = \phi(t) + \tan^{-1}\left[\frac{r(t)}{A_c}\sin[\psi(t)-\phi(t)]\right]$$

$$\theta(t) \simeq \underbrace{\phi(t)}_{\text{signal}} + \underbrace{\frac{r(t)\sin\psi(t)}{A_c}}_{\text{noise}}$$

$\because \tan^{-1}\alpha = \alpha$ when α is small

$$\theta(t) = \underbrace{K_p\, m(t)}_{P_o} + \underbrace{\frac{n_Q(t)}{A_c}}_{P_n}$$

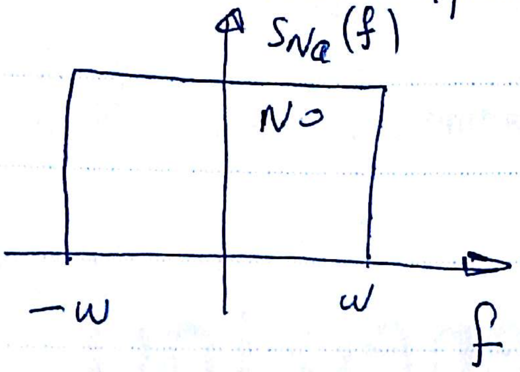
$\because\ r(t)\sin\psi(t) = n_Q(t)$

المركبه العموديه للـ noise

$$\boxed{\therefore\ P_o = K_p^2\, P}$$

مع ملاحظة انه لحساب ال noise power لم يكون هنا
عمليه تفاضل مثل ما حدث في ال FM وكمان سوف
نأخذ ال noise ع طول

+ we Calculate $P_{n_{o/p}}$ from $\left(\frac{n_Q(t)}{A_c^2}\right)$ where



$$S_{N_Q}(f) = \begin{cases} N_o & |f| \le w \\ 0 & \text{otherwise} \end{cases}$$

$$\boxed{\therefore P_{n_{o/p}} = P_{n_Q} = \frac{2N_o w}{A_c^2}} \Rightarrow (2)$$

$$\boxed{\therefore (SNR)_{o/p} = \frac{K_p^2 P A_c^2}{2N_o w}}$$

$$\therefore FoM_{PM} = \frac{(SNR)_{o/p}}{(SNR)_{i/p}} = \frac{K_p^2 P \cancel{A_c^2} / \cancel{2N_o w}}{\cancel{A_c^2} / \cancel{2N_o w}}$$

$$\boxed{FoM_{PM} = K_p^2 P}$$

$$\therefore FOM_{PM} = K_p^2 P$$

(c) for single tone Modulation

$$m(t) = A_m \cos 2\pi f_m t$$

$$\therefore P = \frac{A_m^2}{2}$$

$$\therefore FOM_{PM} = K_p^2 P = K_p^2 \frac{A_m^2}{2} = \frac{1}{2} K_p^2 A_m^2$$

But $\beta_{PM} = K_p \cdot A_m$

$$\boxed{\therefore FOM_{PM\,\text{single tone}} = \frac{1}{2} \beta_{PM}^2}$$

For FM

$$FOM_{FM} = \frac{3}{2} \beta_{FM}^2$$

$$\therefore FOM_{FM} = 3 \text{ times } FOM_{PM}$$

Threshold Effect $\begin{cases} AM \\ FM \end{cases}$

* وهو العيب الثاني في دوائر ال AM & FM ويحدث عندما
تكون ال noise power أعلى من ال Carrier power
وبالتالي فإن ال Envelope Detector لم يستطيع إخراج
ال m(t) بشكل صحيح.

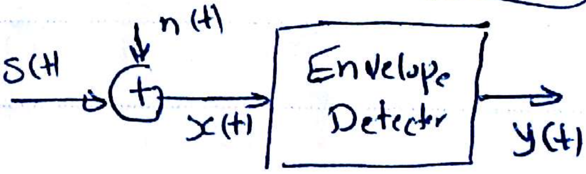
*

* يجب أن تكون ال Carrier power أعلى من ال noise power
بمقدار أكبر من threshold value حتى نستطيع الحصول على
الإشارة m(t) بنجاح. ولكن سوف ندرس الحالة اللتي يكون ال
noise أعلى من ال Carrier

let $$s_{AM}(t) = A_c[1 + k_a m(t)]\cos 2\pi f_c t$$

$$n(t) = r(t)\cos[2\pi f_c t + \psi(t)]$$ ← (noise > Carrier)

$$x(t) = \underbrace{s(t)}_{\frac{A_c^2}{2}} + \underbrace{n(t)}_{2N_0W}$$

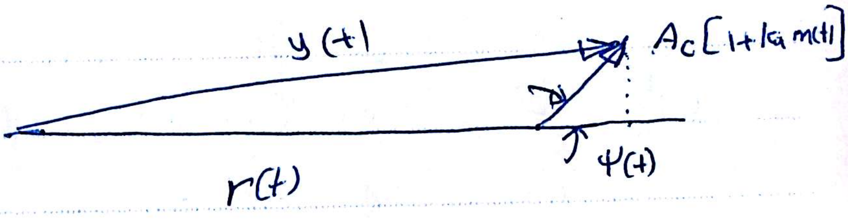


At the envelope i/p :-

$$\boxed{\text{Carrier-to-Noise ratio} = \frac{A_c^2/2}{2N_0W} = \frac{A_c^2}{4N_0W} = \rho \qquad \therefore \rho = \frac{A_c^2}{4N_0W}}$$

At envelop o/p: Using Phasor diagram

مع أخذ r(t) هي ال refrence لأنها هي الكبيرة



$$y(t) \cong r(t) + A_c[1+k_a m(t)]\cos\psi(t) + A_c[1+k_a m(t)]\sin\psi(t)$$

نهمل

$$\boxed{y(t) \cong r(t) + A_c\cos\psi(t) + A_c k_a m(t)\cos\psi(t)}$$

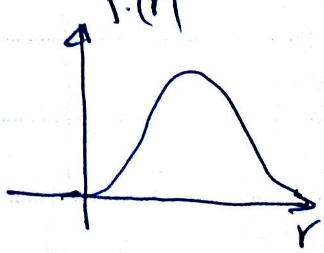
* نلاحظ أن الجزء y(t) لا يحتوي على مركبة صريحة لل m(t)
وإنما تحتوي على ال m(t) مضروبة في $\cos\psi(t)$ التي تمثل ال noise
وبالتالي لن نستطيع الحصول على m(t) بدون ال noise

* وبالتالي يجب أن تكون P كبيرة وأكبر من ال threshold لتفادي
تلك المشكلة والمطلوب معرفة احتمالية أن $P_{Carrier}$ أكبر من ال P_{noise}

* مع ملاحظة أن ال noise عبارة عن متغير عشوائي وله
probability density function of Random Variable عليه نرسم

$$P_{R_t}(r) = \frac{r}{\sigma_N^2}\exp\left(-\frac{r^2}{2\sigma_N^2}\right)$$

$$\boxed{\sigma_N^2 = 2N_0W}$$ Varians



Rayleigh Distribution

$$\therefore\ P_{R_t}(r) = \frac{r}{\sigma_N^2} \exp\left(-\frac{r^2}{2\sigma_N^2}\right)$$

$$P_{R_t}(r) = \frac{r}{2N_0 w} \exp\left(-\frac{r^2}{4N_0 w}\right)$$

and

$$\boxed{P(R_t \geq A_c) = \int_{A_c}^{\infty} P_{R_t}(r) \cdot dr}$$

حفظ

probability

احتمال

$$P(R_t \geq A_c) = \int_{A_c}^{\infty} \frac{r}{2N_0 w} \exp\left(-\frac{r^2}{4N_0 w}\right) dr$$

$$= \exp\left(-\frac{A_c^2}{4N_0 w}\right) = \exp(-\rho)$$

احتمال ان ال noise اعلى من ال Carrier

Carrier to noise ratio

if $\rho = -1.6\,dB$ → envelop detector is well in performance

$\rho = 6.6\,dB$ → " " like Ideal Case
بدون noise

$\rho \leq -1.6\,dB$ → We Cannot restore $m(t)$
ما تطلع اشارة $m(t)$

// Below threshold: system performance is very bad ☹

Ex

$\Rightarrow$ if $P(R_t \geq A_c) = 0.5$

$\therefore \exp(-\rho) = 0.5$

$\therefore \rho = -\ln(0.5) = 0.69$

if $P(R_t \geq A_c) = 0.01$

$\therefore \rho = -\ln(0.01) = 4.6$